

**FACULTY OF ARTS, HUMANITIES & SOCIAL SCIENCES**

**Trinity Business School**

**MSc Business Analytics**

**2021/22**  **Semester 2 2022**

**Annual Examination**

**BU7148 Operations Analytics**

**4th May 2022**  **Online Examination** **09.30am – 11.30am**

**Shivam Gupta (External Examiner)**

**Konstantinos Stouras (Lecturer)**

**Instructions to Candidates**:

The examination is TWO hours in duration **+ 15 min Blackboard upload time**

* This is an individual exam. You are responsible for the academic integrity of your submission.
* Distribution tables are available in the end of the exam paper.
* There are several questions with sub-questions. Answer **ALL** questions and all sub-questions by typing your answer below each respective (sub)question. Please note the weighting of marks for each question.
* If anything is ambiguous to you, please state your assumption(s) and submit your answer according to your best interpretation of the question.
* Save your exam using your Student ID and submit it as a single PDF file on Blackboard. E.g. if your Student ID is **123456** you should submit a PDF file named **123456.pdf**.
* Do not overthink!

**Please provide your Student ID Here:**

**I. Business Intuition** (50 marks in total)

(Q1) Explain why we should always fully utilize the bottleneck of a process. Further, in order to increase the total output of a process would you recommend implementing yield improvements at steps before, at, or after the process’ bottleneck resource? Why? You may use examples to illustrate your answers. (9 marks)

**Answer**:

(Q2) Initially, COVID19 vaccination was taking place in two distinct vaccination centers A and B over three doses as follows. People between 30-50 (respectively, 60-80) years old were assigned to vaccination center A (respectively, B) and each age group respectively had to visit their assigned center for all three doses at pre-specified dates. That is, initially for each population group getting vaccinated at the other vaccination center was not possible. However, at a later stage an operational change has been implemented in which each individual can get vaccinated at any of the two vaccination centers s/he wants. All else equal, briefly explain with reference to a point discussed in class whether and why you agree with the following statements concerning this operational change:

a. “The expected number of people queueing to get vaccinated will increase”.

b. “The expected number of people actually with a nurse receiving the vaccine may increase or may decrease”.

c. “On average, a nurse working either on center A or on center B will become busier”.

d. “The coefficient of variation of arrivals will decrease”. (16 marks)

**Answer**:

(Q3) Suppose an e-commerce retail company (such as Amazon.com) decides to reduce the number of its warehouses from two to one while maintaining the same level of service.

a. Briefly explain how will such a decision most likely impact (i) the transportation costs of products to consumers, and (ii) the amount of inventory stocked? (Will they increase, decrease, or stay the same? Why?)

b. Provide a business setting or industry in which *decentralizing* warehouses would make business sense. Explain why with reference to concepts discussed in class. (10 marks)

**Answer**:

(Q4) Cisco Systems, the market leader in the networking equipment industry, has implemented a world class reverse logistics program. As a part of this program, retailers are offered a rebate on unsold inventory at the end of each quarter. The retailers sell unsold equipment at the price set by Cisco systems (i.e., they have no pricing power). However, based on the rebate the retailers can decide the order quantity. Explain how the result of this rebate will impact retailers’ cost of overage and cost of underage (i.e., will they increase/decrease/remain the same), and whether retailers’ should order a smaller/larger/the same quantity from Cisco. (15 marks)

**Answer**:

**II. Quantitative Questions**  (50 marks in total)

1. **Whole-Mart**  (24 marks in total)

Whole-Mart is a company with several physical stores spread across the US. One of such stores is located in the suburbs of Chicago. Every year (during the week preceding the Thanksgiving Day) this store traditionally attracts thousands of locals who are willing to buy Whole-Mart’s delicious turkey. However Whole-Mart faces a major problem of extremely long waiting times with the length of the queues at the cashier desk estimated to be 6 people (excluding the person being currently serviced). Due to the small size of the building, there is a single cashier desk currently installed in the store.

Check out is a highly variable process (some people are buying turkey only whereas the others are stocking up for the entire period of Christmas holidays). On average, the cashier spends 4 minutes to service a single customer. Also, on average there is a 4.5-minute interval in between arrivals of customers. This time is distributed exponentially.

1. Calculate the current waiting time in the queue. (6 marks)

**Answer**: ­32

1. Calculate the standard deviation of the check-out process. (6 marks)

**Answer**: 4

Frustrated by the long queues, Whole-Mart’s management decides to expand the store’s area and install 3 new cashier desks to help the existing one.

1. Assuming everything else remains the same, calculate the new waiting time and the total time that a customer spends in the queue to get checked out? (6 marks)

**Answer**: 0,024 and 4,024

Hiring three new cashiers works out to be too expensive and Whole-Mart decides to go back to the previous business model with a single cashier desk. The CEO of Whole-Mart heard the saying that “variability is evil” and decides to take a different approach by selling only turkey on the eve of Thanksgiving Day. This step simplifies the process of the check-out and makes the time spent on check-out to be the same for every customer: 1 minute.

1. Based on this model, how long will an average customer have to wait in line before s/he gets to the cashier desk? (6 marks)

**Answer**: 0,24

1. **YOLO**  (26 marks in total)

YOLO.com makes cool headphones and is based in the [beautiful Greek island of Ikaria](https://www.google.com/search?q=ikaria+island+beaches&sxsrf=ALeKk03jlMUVeAMVKnxZWM6bdhghJhyYlA:1619818004156&source=lnms&tbm=isch&sa=X&ved=2ahUKEwjh7fOl9KbwAhVB_7sIHeKtA-IQ_AUoAnoECAEQBA&biw=2048&bih=963). Due to the recent pandemic, YOLO is concerned about how any additional supply chain disruptions might affect the availability of a magnet which is essential for their headphones. It is the start of February and YOLO must commit to a magnet quantity for delivery in July. Each magnet ordered in February will cost YOLO €40/unit. YOLO estimates that demand for magnets in July at a price of €70/unit will be normally distributed with a mean of 250,000 and a standard deviation of 200,000. If YOLO’s demand in July exceeds its regular order (the amount ordered in February) then YOLO can obtain additional units on the spot market. The spot market price for these magnets in July is estimated to be €50/unit. Magnets not used in July will certainly be used in August. However, magnets ordered in March for August delivery are expected to cost €36/unit. Furthermore, due to physical storage costs and opportunity costs of capital there is a €1/unit cost to hold each magnet from one month to the next.

1. Can we apply the newsvendor model here in order to decide on the optimal number of magnets to order in February? If so, why? (6 marks)

**Answer**:

1. Calculate the optimal number of magnets YOLO should order in February. (10 marks)

**Answer**:

1. Find the optimal expected number of magnets to be obtained through the spot market in July. (10 marks)

**Answer**:

**END OF EXAMINATION**

**PLEASE FIND DISTRIBUTION TABLES OVERLEAF**

**Table A:** Standardised Normal Probability Table

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Z | 0.00 | -0.01 | -0.02 | -0.03 | -0.04 | -0.05 | -0.06 | -0.07 | -0.08 | -0.09 |
| -2.9 | 0.00187 | 0.00181 | 0.00175 | 0.00169 | 0.00164 | 0.00159 | 0.00154 | 0.00149 | 0.00144 | 0.00139 |
| -2.8 | 0.00256 | 0.00248 | 0.00240 | 0.00233 | 0.00226 | 0.00219 | 0.00212 | 0.00205 | 0.00199 | 0.00193 |
| -2.7 | 0.00347 | 0.00336 | 0.00326 | 0.00317 | 0.00307 | 0.00298 | 0.00289 | 0.00280 | 0.00272 | 0.00264 |
| -2.6 | 0.00466 | 0.00453 | 0.00440 | 0.00427 | 0.00415 | 0.00402 | 0.00391 | 0.00379 | 0.00368 | 0.00357 |
| -2.5 | 0.00621 | 0.00604 | 0.00587 | 0.00570 | 0.00554 | 0.00539 | 0.00523 | 0.00508 | 0.00494 | 0.00480 |
| -2.4 | 0.00820 | 0.00798 | 0.00776 | 0.00755 | 0.00734 | 0.00714 | 0.00695 | 0.00676 | 0.00657 | 0.00639 |
| -2.3 | 0.01072 | 0.01044 | 0.01017 | 0.00990 | 0.00964 | 0.00939 | 0.00914 | 0.00889 | 0.00866 | 0.00842 |
| -2.2 | 0.01390 | 0.01355 | 0.01321 | 0.01287 | 0.01255 | 0.01222 | 0.01191 | 0.01160 | 0.01130 | 0.01101 |
| -2.1 | 0.01786 | 0.01743 | 0.01700 | 0.01659 | 0.01618 | 0.01578 | 0.01539 | 0.01500 | 0.01463 | 0.01426 |
| -2.0 | 0.02275 | 0.02222 | 0.02169 | 0.02118 | 0.02068 | 0.02018 | 0.01970 | 0.01923 | 0.01876 | 0.01831 |
| -1.9 | 0.02872 | 0.02807 | 0.02743 | 0.02680 | 0.02619 | 0.02559 | 0.02500 | 0.02442 | 0.02385 | 0.02330 |
| -1.8 | 0.03593 | 0.03515 | 0.03438 | 0.03362 | 0.03288 | 0.03216 | 0.03144 | 0.03074 | 0.03005 | 0.02938 |
| -1.7 | 0.04457 | 0.04363 | 0.04272 | 0.04182 | 0.04093 | 0.04006 | 0.03920 | 0.03836 | 0.03754 | 0.03673 |
| -1.6 | 0.05480 | 0.05370 | 0.05262 | 0.05155 | 0.05050 | 0.04947 | 0.04846 | 0.04746 | 0.04648 | 0.04551 |
| -1.5 | 0.06681 | 0.06552 | 0.06426 | 0.06301 | 0.06178 | 0.06057 | 0.05938 | 0.05821 | 0.05705 | 0.05592 |
| -1.4 | 0.08076 | 0.07927 | 0.07780 | 0.07636 | 0.07493 | 0.07353 | 0.07215 | 0.07078 | 0.06944 | 0.06811 |
| -1.3 | 0.09680 | 0.09510 | 0.09342 | 0.09176 | 0.09012 | 0.08851 | 0.08691 | 0.08534 | 0.08379 | 0.08226 |
| -1.2 | 0.11507 | 0.11314 | 0.11123 | 0.10935 | 0.10749 | 0.10565 | 0.10383 | 0.10204 | 0.10027 | 0.09853 |
| -1.1 | 0.13567 | 0.13350 | 0.13136 | 0.12924 | 0.12714 | 0.12507 | 0.12302 | 0.12100 | 0.11900 | 0.11702 |
| -1.0 | 0.15866 | 0.15625 | 0.15386 | 0.15151 | 0.14917 | 0.14686 | 0.14457 | 0.14231 | 0.14007 | 0.13786 |
| -0.9 | 0.18406 | 0.18141 | 0.17879 | 0.17619 | 0.17361 | 0.17106 | 0.16853 | 0.16602 | 0.16354 | 0.16109 |
| -0.8 | 0.21186 | 0.20897 | 0.20611 | 0.20327 | 0.20045 | 0.19766 | 0.19489 | 0.19215 | 0.18943 | 0.18673 |
| -0.7 | 0.24196 | 0.23885 | 0.23576 | 0.23270 | 0.22965 | 0.22663 | 0.22363 | 0.22065 | 0.21770 | 0.21476 |
| -0.6 | 0.27425 | 0.27093 | 0.26763 | 0.26435 | 0.26109 | 0.25785 | 0.25463 | 0.25143 | 0.24825 | 0.24510 |
| -0.5 | 0.30854 | 0.30503 | 0.30153 | 0.29806 | 0.29460 | 0.29116 | 0.28774 | 0.28434 | 0.28096 | 0.27760 |
| -0.4 | 0.34458 | 0.34090 | 0.33724 | 0.33360 | 0.32997 | 0.32636 | 0.32276 | 0.31918 | 0.31561 | 0.31207 |
| -0.3 | 0.38209 | 0.37828 | 0.37448 | 0.37070 | 0.36693 | 0.36317 | 0.35942 | 0.35569 | 0.35197 | 0.34827 |
| -0.2 | 0.42074 | 0.41683 | 0.41294 | 0.40905 | 0.40517 | 0.40129 | 0.39743 | 0.39358 | 0.38974 | 0.38591 |
| -0.1 | 0.46017 | 0.45620 | 0.45224 | 0.44828 | 0.44433 | 0.44038 | 0.43644 | 0.43251 | 0.42858 | 0.42465 |
| 0.0 | 0.50000 | 0.49601 | 0.49202 | 0.48803 | 0.48405 | 0.48006 | 0.47608 | 0.47210 | 0.46812 | 0.46414 |
| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.50000 | 0.50399 | 0.50798 | 0.51197 | 0.51595 | 0.51994 | 0.52392 | 0.52790 | 0.53188 | 0.53586 |
| 0.1 | 0.53983 | 0.54380 | 0.54776 | 0.55172 | 0.55567 | 0.55962 | 0.56356 | 0.56749 | 0.57142 | 0.57535 |
| 0.2 | 0.57926 | 0.58317 | 0.58706 | 0.59095 | 0.59483 | 0.59871 | 0.60257 | 0.60642 | 0.61026 | 0.61409 |
| 0.3 | 0.61791 | 0.62172 | 0.62552 | 0.62930 | 0.63307 | 0.63683 | 0.64058 | 0.64431 | 0.64803 | 0.65173 |
| 0.4 | 0.65542 | 0.65910 | 0.66276 | 0.66640 | 0.67003 | 0.67364 | 0.67724 | 0.68082 | 0.68439 | 0.68793 |
| 0.5 | 0.69146 | 0.69497 | 0.69847 | 0.70194 | 0.70540 | 0.70884 | 0.71226 | 0.71566 | 0.71904 | 0.72240 |
| 0.6 | 0.72575 | 0.72907 | 0.73237 | 0.73565 | 0.73891 | 0.74215 | 0.74537 | 0.74857 | 0.75175 | 0.75490 |
| 0.7 | 0.75804 | 0.76115 | 0.76424 | 0.76730 | 0.77035 | 0.77337 | 0.77637 | 0.77935 | 0.78230 | 0.78524 |
| 0.8 | 0.78814 | 0.79103 | 0.79389 | 0.79673 | 0.79955 | 0.80234 | 0.80511 | 0.80785 | 0.81057 | 0.81327 |
| 0.9 | 0.81594 | 0.81859 | 0.82121 | 0.82381 | 0.82639 | 0.82894 | 0.83147 | 0.83398 | 0.83646 | 0.83891 |
| 1.0 | 0.84134 | 0.84375 | 0.84614 | 0.84849 | 0.85083 | 0.85314 | 0.85543 | 0.85769 | 0.85993 | 0.86214 |
| 1.1 | 0.86433 | 0.86650 | 0.86864 | 0.87076 | 0.87286 | 0.87493 | 0.87698 | 0.87900 | 0.88100 | 0.88298 |
| 1.2 | 0.88493 | 0.88686 | 0.88877 | 0.89065 | 0.89251 | 0.89435 | 0.89617 | 0.89796 | 0.89973 | 0.90147 |
| 1.3 | 0.90320 | 0.90490 | 0.90658 | 0.90824 | 0.90988 | 0.91149 | 0.91309 | 0.91466 | 0.91621 | 0.91774 |
| 1.4 | 0.91924 | 0.92073 | 0.92220 | 0.92364 | 0.92507 | 0.92647 | 0.92785 | 0.92922 | 0.93056 | 0.93189 |
| 1.5 | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 | 0.93943 | 0.94062 | 0.94179 | 0.94295 | 0.94408 |
| 1.6 | 0.94520 | 0.94630 | 0.94738 | 0.94845 | 0.94950 | 0.95053 | 0.95154 | 0.95254 | 0.95352 | 0.95449 |
| 1.7 | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.96080 | 0.96164 | 0.96246 | 0.96327 |
| 1.8 | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 | 0.96784 | 0.96856 | 0.96926 | 0.96995 | 0.97062 |
| 1.9 | 0.97128 | 0.97193 | 0.97257 | 0.97320 | 0.97381 | 0.97441 | 0.97500 | 0.97558 | 0.97615 | 0.97670 |
| 2.0 | 0.97725 | 0.97778 | 0.97831 | 0.97882 | 0.97932 | 0.97982 | 0.98030 | 0.98077 | 0.98124 | 0.98169 |
| 2.1 | 0.98214 | 0.98257 | 0.98300 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.98500 | 0.98537 | 0.98574 |
| 2.2 | 0.98610 | 0.98645 | 0.98679 | 0.98713 | 0.98745 | 0.98778 | 0.98809 | 0.98840 | 0.98870 | 0.98899 |
| 2.3 | 0.98928 | 0.98956 | 0.98983 | 0.99010 | 0.99036 | 0.99061 | 0.99086 | 0.99111 | 0.99134 | 0.99158 |
| 2.4 | 0.99180 | 0.99202 | 0.99224 | 0.99245 | 0.99266 | 0.99286 | 0.99305 | 0.99324 | 0.99343 | 0.99361 |
| 2.5 | 0.99379 | 0.99396 | 0.99413 | 0.99430 | 0.99446 | 0.99461 | 0.99477 | 0.99492 | 0.99506 | 0.99520 |
| 2.6 | 0.99534 | 0.99547 | 0.99560 | 0.99573 | 0.99585 | 0.99598 | 0.99609 | 0.99621 | 0.99632 | 0.99643 |
| 2.7 | 0.99653 | 0.99664 | 0.99674 | 0.99683 | 0.99693 | 0.99702 | 0.99711 | 0.99720 | 0.99728 | 0.99736 |
| 2.8 | 0.99744 | 0.99752 | 0.99760 | 0.99767 | 0.99774 | 0.99781 | 0.99788 | 0.99795 | 0.99801 | 0.99807 |
| 2.9 | 0.99813 | 0.99819 | 0.99825 | 0.99831 | 0.99836 | 0.99841 | 0.99846 | 0.99851 | 0.99856 | 0.99861 |

**Table B:** Standardised Normal Loss Table

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| z | 0.00 | -0.01 | -0.02 | -0.03 | -0.04 | -0.05 | -0.06 | -0.07 | -0.08 | -0.09 |
| -2.9 | 2.90054 | 2.91052 | 2.92051 | 2.93049 | 2.94047 | 2.95046 | 2.96044 | 2.97042 | 2.98041 | 2.99040 |
| -2.8 | 2.80076 | 2.81074 | 2.82071 | 2.83069 | 2.84066 | 2.85064 | 2.86062 | 2.87060 | 2.88058 | 2.89056 |
| -2.7 | 2.70106 | 2.71103 | 2.72099 | 2.73096 | 2.74093 | 2.75090 | 2.76087 | 2.77084 | 2.78081 | 2.79079 |
| -2.6 | 2.60146 | 2.61142 | 2.62137 | 2.63133 | 2.64129 | 2.65125 | 2.66121 | 2.67117 | 2.68113 | 2.69110 |
| -2.5 | 2.50200 | 2.51194 | 2.52188 | 2.53183 | 2.54177 | 2.55171 | 2.56166 | 2.57161 | 2.58156 | 2.59151 |
| -2.4 | 2.40272 | 2.41264 | 2.42256 | 2.43248 | 2.44241 | 2.45234 | 2.46227 | 2.47220 | 2.48213 | 2.49207 |
| -2.3 | 2.30366 | 2.31356 | 2.32345 | 2.33335 | 2.34325 | 2.35316 | 2.36307 | 2.37298 | 2.38289 | 2.39280 |
| -2.2 | 2.20489 | 2.21475 | 2.22462 | 2.23449 | 2.24436 | 2.25423 | 2.26411 | 2.27400 | 2.28388 | 2.29377 |
| -2.1 | 2.10647 | 2.11629 | 2.12612 | 2.13595 | 2.14579 | 2.15563 | 2.16547 | 2.17532 | 2.18517 | 2.19503 |
| -2.0 | 2.00849 | 2.01827 | 2.02805 | 2.03783 | 2.04762 | 2.05742 | 2.06722 | 2.07702 | 2.08683 | 2.09665 |
| -1.9 | 1.91105 | 1.92077 | 1.93049 | 1.94022 | 1.94996 | 1.95970 | 1.96945 | 1.97920 | 1.98896 | 1.99872 |
| -1.8 | 1.81428 | 1.82392 | 1.83357 | 1.84323 | 1.85290 | 1.86257 | 1.87226 | 1.88195 | 1.89164 | 1.90134 |
| -1.7 | 1.71829 | 1.72785 | 1.73742 | 1.74699 | 1.75658 | 1.76617 | 1.77578 | 1.78539 | 1.79501 | 1.80464 |
| -1.6 | 1.62324 | 1.63270 | 1.64217 | 1.65165 | 1.66114 | 1.67064 | 1.68015 | 1.68967 | 1.69920 | 1.70874 |
| -1.5 | 1.52931 | 1.53865 | 1.54800 | 1.55736 | 1.56674 | 1.57612 | 1.58552 | 1.59494 | 1.60436 | 1.61380 |
| -1.4 | 1.43667 | 1.44587 | 1.45508 | 1.46431 | 1.47356 | 1.48281 | 1.49208 | 1.50137 | 1.51067 | 1.51998 |
| -1.3 | 1.34553 | 1.35457 | 1.36363 | 1.37270 | 1.38179 | 1.39090 | 1.40002 | 1.40916 | 1.41831 | 1.42748 |
| -1.2 | 1.25610 | 1.26496 | 1.27384 | 1.28274 | 1.29165 | 1.30059 | 1.30954 | 1.31851 | 1.32750 | 1.33650 |
| -1.1 | 1.16862 | 1.17727 | 1.18595 | 1.19465 | 1.20336 | 1.21210 | 1.22086 | 1.22964 | 1.23844 | 1.24726 |
| -1.0 | 1.08332 | 1.09174 | 1.10019 | 1.10866 | 1.11716 | 1.12568 | 1.13422 | 1.14279 | 1.15138 | 1.15999 |
| -0.9 | 1.00043 | 1.00860 | 1.01680 | 1.02503 | 1.03328 | 1.04156 | 1.04986 | 1.05819 | 1.06654 | 1.07491 |
| -0.8 | 0.92021 | 0.92810 | 0.93603 | 0.94398 | 0.95196 | 0.95997 | 0.96801 | 0.97607 | 0.98417 | 0.99229 |
| -0.7 | 0.84288 | 0.85048 | 0.85810 | 0.86576 | 0.87345 | 0.88117 | 0.88892 | 0.89669 | 0.90450 | 0.91234 |
| -0.6 | 0.76867 | 0.77595 | 0.78325 | 0.79059 | 0.79797 | 0.80537 | 0.81281 | 0.82028 | 0.82778 | 0.83531 |
| -0.5 | 0.69780 | 0.70473 | 0.71170 | 0.71870 | 0.72573 | 0.73281 | 0.73991 | 0.74705 | 0.75422 | 0.76143 |
| -0.4 | 0.63044 | 0.63701 | 0.64362 | 0.65027 | 0.65695 | 0.66367 | 0.67042 | 0.67721 | 0.68404 | 0.69090 |
| -0.3 | 0.56676 | 0.57296 | 0.57920 | 0.58547 | 0.59178 | 0.59813 | 0.60452 | 0.61094 | 0.61740 | 0.62390 |
| -0.2 | 0.50689 | 0.51271 | 0.51856 | 0.52445 | 0.53038 | 0.53634 | 0.54235 | 0.54840 | 0.55448 | 0.56060 |
| -0.1 | 0.45094 | 0.45635 | 0.46181 | 0.46731 | 0.47285 | 0.47842 | 0.48404 | 0.48969 | 0.49539 | 0.50112 |
| 0.0 | 0.39894 | 0.40396 | 0.40902 | 0.41412 | 0.41926 | 0.42444 | 0.42966 | 0.43492 | 0.44022 | 0.44556 |
| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.39894 | 0.39396 | 0.38902 | 0.38412 | 0.37926 | 0.37444 | 0.36966 | 0.36492 | 0.36022 | 0.35556 |
| 0.1 | 0.35094 | 0.34635 | 0.34181 | 0.33731 | 0.33285 | 0.32842 | 0.32404 | 0.31969 | 0.31539 | 0.31112 |
| 0.2 | 0.30689 | 0.30271 | 0.29856 | 0.29445 | 0.29038 | 0.28634 | 0.28235 | 0.27840 | 0.27448 | 0.27060 |
| 0.3 | 0.26676 | 0.26296 | 0.25920 | 0.25547 | 0.25178 | 0.24813 | 0.24452 | 0.24094 | 0.23740 | 0.23390 |
| 0.4 | 0.23044 | 0.22701 | 0.22362 | 0.22027 | 0.21695 | 0.21367 | 0.21042 | 0.20721 | 0.20404 | 0.20090 |
| 0.5 | 0.19780 | 0.19473 | 0.19170 | 0.18870 | 0.18573 | 0.18281 | 0.17991 | 0.17705 | 0.17422 | 0.17143 |
| 0.6 | 0.16867 | 0.16595 | 0.16325 | 0.16059 | 0.15797 | 0.15537 | 0.15281 | 0.15028 | 0.14778 | 0.14531 |
| 0.7 | 0.14288 | 0.14048 | 0.13810 | 0.13576 | 0.13345 | 0.13117 | 0.12892 | 0.12669 | 0.12450 | 0.12234 |
| 0.8 | 0.12021 | 0.11810 | 0.11603 | 0.11398 | 0.11196 | 0.10997 | 0.10801 | 0.10607 | 0.10417 | 0.10229 |
| 0.9 | 0.10043 | 0.09860 | 0.09680 | 0.09503 | 0.09328 | 0.09156 | 0.08986 | 0.08819 | 0.08654 | 0.08491 |
| 1.0 | 0.08332 | 0.08174 | 0.08019 | 0.07866 | 0.07716 | 0.07568 | 0.07422 | 0.07279 | 0.07138 | 0.06999 |
| 1.1 | 0.06862 | 0.06727 | 0.06595 | 0.06465 | 0.06336 | 0.06210 | 0.06086 | 0.05964 | 0.05844 | 0.05726 |
| 1.2 | 0.05610 | 0.05496 | 0.05384 | 0.05274 | 0.05165 | 0.05059 | 0.04954 | 0.04851 | 0.04750 | 0.04650 |
| 1.3 | 0.04553 | 0.04457 | 0.04363 | 0.04270 | 0.04179 | 0.04090 | 0.04002 | 0.03916 | 0.03831 | 0.03748 |
| 1.4 | 0.03667 | 0.03587 | 0.03508 | 0.03431 | 0.03356 | 0.03281 | 0.03208 | 0.03137 | 0.03067 | 0.02998 |
| 1.5 | 0.02931 | 0.02865 | 0.02800 | 0.02736 | 0.02674 | 0.02612 | 0.02552 | 0.02494 | 0.02436 | 0.02380 |
| 1.6 | 0.02324 | 0.02270 | 0.02217 | 0.02165 | 0.02114 | 0.02064 | 0.02015 | 0.01967 | 0.01920 | 0.01874 |
| 1.7 | 0.01829 | 0.01785 | 0.01742 | 0.01699 | 0.01658 | 0.01617 | 0.01578 | 0.01539 | 0.01501 | 0.01464 |
| 1.8 | 0.01428 | 0.01392 | 0.01357 | 0.01323 | 0.01290 | 0.01257 | 0.01226 | 0.01195 | 0.01164 | 0.01134 |
| 1.9 | 0.01105 | 0.01077 | 0.01049 | 0.01022 | 0.00996 | 0.00970 | 0.00945 | 0.00920 | 0.00896 | 0.00872 |
| 2.0 | 0.00849 | 0.00827 | 0.00805 | 0.00783 | 0.00762 | 0.00742 | 0.00722 | 0.00702 | 0.00683 | 0.00665 |
| 2.1 | 0.00647 | 0.00629 | 0.00612 | 0.00595 | 0.00579 | 0.00563 | 0.00547 | 0.00532 | 0.00517 | 0.00503 |
| 2.2 | 0.00489 | 0.00475 | 0.00462 | 0.00449 | 0.00436 | 0.00423 | 0.00411 | 0.00400 | 0.00388 | 0.00377 |
| 2.3 | 0.00366 | 0.00356 | 0.00345 | 0.00335 | 0.00325 | 0.00316 | 0.00307 | 0.00298 | 0.00289 | 0.00280 |
| 2.4 | 0.00272 | 0.00264 | 0.00256 | 0.00248 | 0.00241 | 0.00234 | 0.00227 | 0.00220 | 0.00213 | 0.00207 |
| 2.5 | 0.00200 | 0.00194 | 0.00188 | 0.00183 | 0.00177 | 0.00171 | 0.00166 | 0.00161 | 0.00156 | 0.00151 |
| 2.6 | 0.00146 | 0.00142 | 0.00137 | 0.00133 | 0.00129 | 0.00125 | 0.00121 | 0.00117 | 0.00113 | 0.00110 |
| 2.7 | 0.00106 | 0.00103 | 0.00099 | 0.00096 | 0.00093 | 0.00090 | 0.00087 | 0.00084 | 0.00081 | 0.00079 |
| 2.8 | 0.00076 | 0.00074 | 0.00071 | 0.00069 | 0.00066 | 0.00064 | 0.00062 | 0.00060 | 0.00058 | 0.00056 |
| 2.9 | 0.00054 | 0.00052 | 0.00051 | 0.00049 | 0.00047 | 0.00046 | 0.00044 | 0.00042 | 0.00041 | 0.00040 |

**Table C:** *Lq* Values for the Multi-server Queue

Values of *Lq* for *s* servers, with mean utilization rate **, assuming Poisson arrivals and exponential service times.

Utilization rate Number of servers (s)

 1 2 3 4 5

.10 .0111 .0020 .0004 .0001 .0000

.20 .0500 .0167 .0062 .0024 .0010

.30 .1286 .0593 .0300 .0159 .0086

.35 .1885 .0977 .0552 .0325 .0196

.40 .2667 .1524 .0941 .0605 .0398

.45 .3682 .2285 .1522 .1052 .0743

.50 .5000 .3333 .2368 .1739 .1304

.55 .6722 .4771 .3583 .2772 .2185

.60 .9000 .6750 .5321 .4306 .3542

.62 1.0116 .7743 .6213 .5109 .4269

.64 1.1378 .8880 .7246 .6051 .5130

.66 1.2812 1.0188 .8446 .7158 .6152

.68 1.4450 1.1698 .9847 .8461 .7367

.70 1.6333 1.3451 1.1488 1.0002 .8816

.72 1.8514 1.5500 1.3423 1.1834 1.0553

.74 2.1062 1.7914 1.5721 1.4025 1.2646

.76 2.4067 2.0785 1.8472 1.6668 1.5187

.78 2.7655 2.4237 2.1803 1.9887 1.8302

.80 3.2000 2.8444 2.5888 2.3857 2.2165

.82 3.7356 3.3661 3.0979 2.8832 2.7029

.84 4.4100 4.0265 3.7456 3.5190 3.3273

.86 5.2829 4.8852 4.5914 4.3526 4.1493

.88 6.4533 6.0414 5.7345 5.3834 5.2682

.90 8.1000 7.6737 7.3535 7.0898 6.8624

.92 10.5800 10.1392 9.8056 9.5290 9.2893

.94 14.7267 14.2712 13.9240 13.6344 13.3821

.96 23.0400 22.5698 22.2088 21.9060 21.6408

.98 48.0200 47.5350 47.1602 46.8439 46.5656

.99 98.0101 97.5176 97.1357 96.8127 96.4274